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Anti-deSitter Vacua of Gauged Supergravities with 8 Supercharges

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Abstract

We investigate supersymmetric extrema of Abelian gauged supergravity theories with non-trivial vector multiplets and 8 supercharges in four and five dimensions. The scalar fields of these models parameterize a manifold consisting of disconnected branches and restricting to the case where this manifold has a non-singular metric we show that on every branch there can be at most one extremum, which is a local maximum (for $W > 0$) or a minimum (for $W < 0$) of the superpotential W. Therefore, these supergravity models do not allow for regular domain wall solutions interpolating between different extrema of the superpotential and the space-time transverse to the wall asymptotically always approaches the boundary of AdS (UV-fixed points in a dual field theory).

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There has been renewed interest in supergravity theories which allow for anti de Sitter (AdS) vacuum solutions. On one hand, the AdS/CFT correspondence implies that domain wall solutions encode the information on the renormalization group (RG) flow of (strongly coupled) super Yang Mills theories as discussed in [1]. On the other hand, in the Randall-Sundrum scenario [2, 3] one considers a domain wall in a five-dimensional AdS space-time which allows for localization of gravity near the wall. In both cases, the gravitational effects in the domain wall backgrounds play an essential role. In the context of fundamental theory it is essential to consider supergravity theories with a non-trivial potential and demonstrate the existence of the domain wall solutions with the desired gravitational effects. The first examples of the superymmetric domain walls were found in N=1 D=4 supergravity theory [4] (for a review see [5]). These solutions are static and interpolate between isolated supersymmetric extrema of the scalar potential. The gravitational properties of these domain walls crucially depend on the features of the superpotential and have been classified in [6, 7].

Unlike supergravity theories with four supercharges, e.g., N=1 D=4 supergravity, which have a rich structure of possible domain walls, the field theory embedding of possible domain wall solutions becomes highly constrained in supergravity theories with at least 8 supercharges, i.e. (N=1, D=5), (N=2, D=4) or (N=4, D=3) supergravity. In these cases the structure of the potential is related to gaugings of isometries of the scalar field manifold and thus it is much more restricted. There are the following different possibilities to gauge these supergravity models: (i) to gauge a subgroup of the $SU(2)$-R-symmetry, (ii) to gauge isometries of the vector moduli space or (iii) to gauge isometries of hyper-multiplet moduli space. In four dimensions the different cases have been reviewed in [8] and the 5-d cases are discussed in [9, 10, 11].

We will show that for the case (i), i.e. for *Abelian gauged* supergravity with eight supercharges, all extrema of the superpotential are disconnected as long as we restrict ourselves to scalar field manifolds with a non-singular metric. It is therefore impossible to construct regular domain wall solutions interpolating between different extrema. In addition, the space-time transverse to the wall always approaches the boundary of AdS asymptotically and thus these solutions disallow for the localization of gravity near the interior of the wall, i.e. in D=5 it is impossible to embed the Randall-Sundrum scenario in this framework. [3]

1) $D = 5$ **Case** For this case equivalent conclusions have been derived in [12]. In D=5 supergravity with 8 supercharges the (real) scalars in the vector multiplets ϕ^A parameterize a hypersurface $\mathcal{M}$ defined by a cubic equation [14]:

$$F(X) \equiv \frac{1}{6} C_{IJK} X^I X^J X^K = 1 \tag{1}$$

where $I = 0, 1, 2, \ldots n$ and n is the number of vector multiplets and C_{IJK} are the coefficient defining the cubic Chern-Simons term in the supergravity Lagrangian. (In

[3] The same conclusion seems to hold also for the case (ii)[12, 13]. On the other hand, we restrict ourselves to studying vector multiplets only and will not consider case (iii).

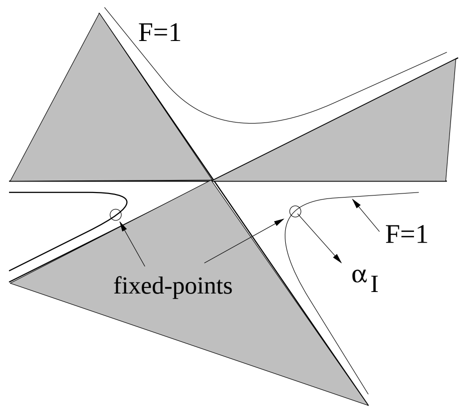

Figure 1: The scalar fields of vector super-multiplets of D=5 theory parameterize a manifold that consists of different branches and due to the attractor equations point where the normal vector is parallel to a given constant vector α_I are "fixed-points" or extrema of the superpotential. The straight lines correspond to F=0 domain and shaded areas to $F < 0$ domains.

Calabi-Yau compactifications these are the topological intersection numbers.) In general $\mathcal{M}$ is not connected and consists of different branches, separated by regions where $F(X) < 0$; see figure 1.

Gauging a $U(1)$ subgroup of the $SU(2)$ R-symmetry [9], the scalars remain uncharged, however they obtain a potential given by

$$V = 6\Big(\frac{3}{4}g^{AB}\partial_A W \partial_B W - W^2\Big) \ , \tag{2}$$

with the superpotential and the metric:

$$W = \alpha_I X^I \ , \quad g_{AB} = -\frac{1}{2}\Big(\partial_A X^I \partial_B X^J \, \partial_I \partial_J F(X)\Big)\Big|_{F=1} \ , \tag{3}$$

From the M/string-theory perspective this superpotential appears due to calibrated sub-manifolds of the internal space, where the vector α_I corresponds to non-trivial fluxes (see [11, 15]). On the other hand, from D=5 perspective this superpotential arises solely due to constraints of supersymmetry [16, 9]. Supersymmetric extrema of V are given by extrema of W and because $\partial_A X^I$ defines tangent vectors on $\mathcal{M}$, supersymmetric extrema are points on $\mathcal{M}$ where the constant vector α_I is normal to $\mathcal{M}$, i.e. where

$$\alpha_I \sim X_I, \tag{4}$$

with $X_I = \frac{1}{3}\partial_I F(X)|_{F=1}$ (see also figure 1). This is the essence of the attractor equation as derived in [17, 18], which has been discussed in the domain wall context in [19, 20].

In addition, the second derivative of W satisfies the following constraint [14]:

$$\partial_A \partial_B W = \frac{2}{3}g_{AB}W + T_{ABC}g^{CE}\partial_E W \ , \tag{5}$$

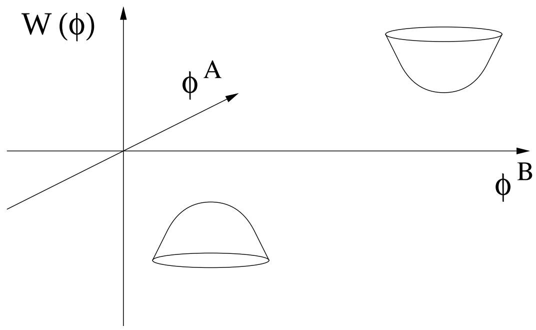

Figure 2: Restricting to a region where the kinetic part of the scalar fields is strictly positive definite, any critical point, where $\partial_A W(\phi^A) = 0$, is either a minimum (for $W > 0$) or a maximum (for $W < 0$). Since saddle points are also excluded, the critical points cannot be connected in a regular way.

where $T_{ABC} \sim \partial_A X^I \partial_B X^J \partial_C X^K C_{IJK}$.

At the extrema of the superpotential (fixed-points) $\partial_E W = 0$ and thus eq. (5) implies that for a positive definite scalar metric g_{AB} these extrema can only be *minima* (for $W > 0$) or *maxima* (for $W < 0$), but *not saddle points*. Moreover this equation implies that the supersymmetric extrema always correspond to the *maxima* of the potential, i.e., $\partial_A \partial_B V = -4 g_{AB} W^2$, see also [21].

Such two extrema, (see figure 2) could be connected if one allows for a saddle point in-between, however since we restricted ourselves to the physical domain of the scalar metric, i.e. we assume that g_{AB} is positive definite, such saddle points are excluded and all the extrema of W lie on disconnected branches of $\mathcal{M}$. [4]

Let us also point out that the space-time transverse to the wall necessarily asymptotes to the boundary of AdS and not the Cauchy AdS horizon, thus exceeding the one which is necessary for implementation of the Randall-Sundrum set-up. (Equivalent arguments are given in [12].) Namely, for the static domain wall Ansatz:

$$ds^2 = \mathcal{A}(z)(-dt^2 + \sum dx_i^2) + dz^2 \tag{6}$$

the Killing spinor equations which fixes the scalars $\phi^A(z)$ take the form [7, 23, 24]:

$$\partial_z \phi^A = \pm 3 g^{AB} \partial_B W \quad , \quad \partial_z \log \mathcal{A} = \mp 2W, \tag{7}$$

with the spinor constraint $\Gamma_z \epsilon = \pm \epsilon$. The expansion of the kink solution $\phi^A = \phi^A_{|\pm} + \delta\phi^A$ around the supersymmetric extremum ($\partial_B W_{|\pm} = 0$) renders (7) in the following asymptotic form:

$$\partial_z (\log \delta\phi^A) = \pm 2 W_{|\pm} \ , \quad \partial_z (\log \mathcal{A}) = \mp 2 W_{|\pm} \ . \tag{8}$$

[4] One arrives at the same conclusion if one assumes that $\mathcal{M}$ is convex, which implies that there can only be one point on any given branch where (4) holds [22].

(In the derivation of the first eq. in (8) we employed (5), evaluated at $\partial_B W = 0$.) The first eq. in (8) implies that for a kink solution to approach (exponentially fast) the asymptotic values $\phi^A_\pm$ (as $z \to \pm\infty$) the superpotential W has to satisfy: $\mathrm{signW}_+ = -\mathrm{signW}_-$. As a consequence, the second eq. in (8) implies that in this case the metric coefficient $\mathcal{A}$ necessarily grows exponentially fast on either side of the wall. Thus, these walls, in addition to being singular, necessarily approach the boundary of the AdS space-time as $z \to \pm\infty$; they have a *repulsive gravity on either side* of the wall and thus *cannot* localize gravity. In a dual field theory these supersymmetric extrema always correspond to ultra-violet (UV) fixed-points [12]. The one-scalar example of such walls were given in [7, 20] (see also [25] for an early work on supergravity kinks); in the interior these walls have a power-law curvature singularity.

Another comment is in order. If one does not insist on the positive definite scalar metric g_{AB}, some supersymmetric extrema can become saddle points[5]. In this case it is possible to connect, e.g., a supersymmetric maximum with a supersymmetric saddle point in a continuous manner. However, again due to (8), the space-time on either side of such non-singular walls asymptotes to the boundaries of AdS, which are UV fixed points of the dual field theories.

2) $D = 4$ **Case** As the second example we consider D=4 , N=2 gauged supergravity (for a review see [8]). In contrast to the D=5 case before, the scalars of vector supermultiplets are now complex and the potential is given by

$$V = e^K \Big(g^{A\bar{B}} D_A W D_{\bar{B}} \bar{W} - 3|W|^2 \Big), \tag{9}$$

where W is the superpotential, K is the Kähler potential, $D_A W \equiv \Big(\partial_A + (\partial_A K)\Big) W$, and $g_{A\bar{B}} = \partial_A \partial_{\bar{B}} K$ is the Kähler metric. In comparison with 5-d supergravity we have to replace the scalars X^I by the symplectic section (X^I, F_I) where $F_I = \partial_I F(X)$ denotes is derivative of the prepotential $F(X)$. The superpotential is again a linear function, but now in the symplectic section [26, 15, 27]:

$$W = \alpha_I X^I - \beta^I F_I \ . \tag{10}$$

Supersymmetric extrema of V are given by extrema of W with respect to the *covarient* derivatives, i.e. $D_A W = 0$. In order to facilitate the investigation of supersymmetric extrema we write the potential in terms of a real function $\widehat{W}$:

$$\widehat{W} \equiv \xi |W| e^{K/2} = \xi |\alpha_I L^I - \beta^I M_I| \ , \tag{11}$$

which is invariant under Kähler transformations and the analogous constraint to (1) becomes $i(\bar{L}^I M_I - L^I \bar{M}_I) = 1$. Here $\xi = \pm 1$ and can only change sign iff W passes through zero.

[5]For special parameter choices the extrema of W may be at the boundary of $\mathcal{M}$ and may not correspond to AdS vacua. We thank S. Gubser for communications on this point.

In terms of $\widehat{W}$, the potential (9) takes the form:

$$V = 3\Big(\frac{4}{3}g^{A\bar{B}}\partial_A\widehat{W}\partial_{\bar{B}}\widehat{W} - \widehat{W}^2\Big). \tag{12}$$

Since $\widehat{W}$ satisfies the relation: $(\partial_A\widehat{W})\widehat{W}^{-1} = (D_AW)(2W)^{-1}$, extrema of $\widehat{W}$ correspond to the supersymmetric extrema of the potential. Note, that employing the real function $\widehat{W}$ the potential (12) has been cast in a form completely parallel to that of D=5 potential. In order to obtain the second derivatives at extrema, we can employ basic formulae from special geometry. Namely, the symplectic section $\mathcal{V} = (L^I, M_I) = e^{K/2}(X^I, F_I)$ satisfies [8]

$$D_AD_B\mathcal{V} = iC_{ABC}g^{C\bar{E}}D_{\bar{E}}\bar{\mathcal{V}} \quad , \quad D_AD_{\bar{B}}\mathcal{V} = g_{A\bar{B}}\mathcal{V} \ , \tag{13}$$

where C_{ABC} is the covariantly holomorphic section. Moreover, using the definition of $\widehat{W}$ one finds for the second derivatives: $(\partial_A\partial_B\widehat{W})\widehat{W}^{-1} = (D_AD_BW)(2W)^{-1} + \mathcal{O}(D_AW)$ and because $D_A\mathcal{V} \equiv (\partial_A + \frac{1}{2}(\partial_AK))\mathcal{V} = e^{K/2}D_AW$ we obtain $(\partial_A\partial_B\widehat{W})\widehat{W}^{-1} = (D_AD_B\mathcal{V})(2\mathcal{V})^{-1} + \mathcal{O}(D_AW)$. As a consequence of (13) we find that at supersymmetric extrema ($\partial_A\widehat{W} = 0$) the second derivatives of $\widehat{W}$ satisfy:

$$\partial_A\partial_{\bar{B}}\widehat{W} = \frac{1}{2}g_{A\bar{B}}\widehat{W} \quad , \quad \partial_A\partial_B\widehat{W} = 0 \ . \tag{14}$$

The relationships (14) imply the same conclusions as in D=5 case: for the domain with a positive definite Kähler metric $g_{A\bar{B}}$, i.e. restricting to the physical region of the metric, all the extrema of $\widehat{W}$ are either minima (for $\widehat{W} > 0$) or maxima (for $\widehat{W} < 0$), but never saddle points. This result again implies that the supersymmetric extrema are disconnected and that the potential always has maxima there, i.e. $\partial_A\partial_{\bar{B}}V = -g_{A\bar{B}}\widehat{W}^2$ and $\partial_A\partial_BV = 0$.

For the purpose of addressing the space-time properties of supersymmetric (static) domain wall backgrounds one arrives at the following Killing spinor equations which we cast in an explicitly Kähler invariant form:

$$\partial_z\phi^A = -2g^{A\bar{B}}\partial_{\bar{B}}\widehat{W} \ , \ \ \partial_z \log \mathcal{A} = \widehat{W} \ . \tag{15}$$

In addition, the complex scalar fields have to statisfy the "geodesic equation":

$$\mathrm{Im}\Big[(\partial_z\phi^A)\partial_A(\log\widehat{W})\Big] = 0 \ , \tag{16}$$

while the Killing spinors satisfy: $\epsilon_\alpha = \xi i\gamma^z e^{\theta_W}\varepsilon_{\alpha\beta}\epsilon^\beta$, where θ_W is the phase of the holomorphic superpotential W. The geodesic eq. (16) is a supergravity generalization of the equation in global supersymmetric theory where a kink solution corresponds to a straight line in the W-plane ($\partial_z\theta_W = 0$) (see, e.g., [28, 29]). The above Killing spinor equations were first derived for domain walls in D=4 N=1 supergravity [4] , where the superpotential W and Kähler potential K are not subject to constraints of N=2

special geometry; of course for the N=1 case the equations remain the same, but with constrained K and W.

The expansion of the kink solution $\phi^A = \phi^A_{|\pm} + \delta\phi^A$ around the supersymmetric extremum ($\partial_B \widehat{W}_{|\pm} = 0$) renders (15) in the following asymptotic form:

$$\partial_z(\log \delta\phi^A) = -2\widehat{W}_\pm \ , \quad \partial_z(\log \mathcal{A}) = 2\widehat{W}_\pm \ . \tag{17}$$

In the derivation of the first eq. in (17) we used the relationships (14). The first eq. in (17) implies that that for the existence of a kink solution ($\phi^A \to \phi^A_{|\pm}$ as $z \to \pm\infty$) the superpotential W necessarily crosses zero and thus $\xi_{|+} = -\xi_{|-}$. The second eq. in (17) in turn implies that in this case the metric coefficient $\mathcal{A}$ necessarily grows exponentially fast on either side of the wall. Thus, just as in the D=5 case, these walls are necessarily singular (because extrema are on different branches) and the space-time asymptotically approaching the boundary of the AdS on either side of the wall (UV fixed points). On the other hand, just as in the D=5 case, if one relaxes the constraint of positive definite Kähler metric, such domains could connect across a smooth region, but the asymptotic space-time remains to approach the boundary of AdS asymptotically.

We have not considered the D=3 case with 8 supercharges, where the corresponding scalars parameterize a quarternionic manifold; we expect, however, the same conclusions. On the other hand, just as for the D=4 cases, breaking further supersymmetry (to four or only two supercharges) one expects a much richer structure (see [30]).

Let us end with some general remarks. In our arguments it was important to assume that the Kähler metric is everywhere positive definite, which excluded saddle points and disconnected all supersymmetric extrema. This is a very strong restriction, which may not be the case for physically interesting applications. E.g., the manifold $\mathcal{M}$ can have boundaries where eigenvalues of the Kähler metric vanish and additional massless modes are expected. In addition, we restricted ourselves to supersymmtric cases only, but it may be that the supersymmetric vacua are connected by non-BPS sphaleron configurations as recently discussed in [31]. These are very interesting aspects, which certainly deserve further investigations.

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